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THE INFLUENCE OF ENVIRONMENTAL NON-STATIONARITY IN A SEQUENTIAL DECISION-MAKING EXPERIMENT

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<u>Summary</u>: ~~This paper report~~s on a series of pilot experiments, and on their theoretical background, that [illegible] ~~were conducted to study the~~ effect on human decision-making of a belief that the environment is changing when in reality it is constant. The results suggest that subjects tend to search more among poorer alternatives when they believe that the situation is changeable, and in conformity with mathematical models suggested by W.K. Estes and R. R. Bush to describe the two types of decision-making behavior. ( )

1. Introduction.

W. K. Estes reported certain experimental results, in a talk at one of the early plenary sessions of the Seminar, that stimulated me both to review some old data of mine and to conduct a new experiment using human subjects in a multiple-choice situation provided by the punchboard demonstrated during my own plenary session talk. The type of result obtained by Estes, that intrigued me, has been reproduced by other investigators in other laboratories both with human subjects and with rats; it is as follows: In certain 2-choice situations where the reward probabilities are π_1 and π_2 the organism will tend eventually to choose the i^{th} alternative with probability

$$p_i^{\infty} = \frac{1-\pi_j}{2-\pi_1-\pi_2} \qquad \text{where } i \neq j.$$

For example, Estes reported a case in which the reward probabilities were $\pi_1 = 1/2$ and $\pi_2 = 0$ and the average tendency of several human subjects was found experimentally to be $p_1^{\infty} = 0.67$. Incidentally, Estes remarked that the formula for p_i^{∞} not only described all his experimental data well, when averaged over enough subjects and trials, but that it was derived from a system of mathematical axioms forming a basis for his learning theory.

During the discussion of the Estes paper it was argued (incorrectly) by some of the game theorists in the audience that such behavior was surprisingly "irrational" since the optimal strategy for the organism is clearly pure rather than mixed, and so the organism should learn eventually to choose only the alternative that provides the more frequent reward. I countered with the following two objections to this game-theoretical argument:

(a) The payoff utilities could not be well defined, as is so unfortunately the usual case in any of our attempts to apply game-theoretic arguments to a real case, and there is a reasonable payoff matrix that would rationalize the reported behavior. Thus, if the organism's object were to get the very best score that it could, rather than simply to maximize its expectation, then it should sometimes not tend to use a pure strategy. I illustrated this case by one of my punchboard experiments in which a very intelligent subject remarked that her only hope to get a "perfect" score in a (90,10) experiment would be to play 90 in the 90-case and 10 in the 10-case and be lucky on each play; in other words, if absolute perfection were assigned sufficiently high utility her optimal behavior even in the game-theoretic sense would not be to always choose the same case. Of course, in a (50,0) experiment this same argument might suggest that she should use a single-choice strategy but only if she is thoroughly convinced that one response will always be non-rewarded. At any rate, a weak defense of the mixed-choice behavior can be made along these general lines.

(b) The von Neumann-Morgenstern game theory is not applicable in this situation unless the organism can safely assume that the experimental stimulus is generated by a stationary stochastic process. For example, if the organism believes that there may be some pattern (non-stationarity) over time in the stimulus, then it can often do better by using a mixed rather than a pure strategy for otherwise it would have no way to discover any pattern effect after the time that it settled on a pure strategy. In fact, the present state of

mathematical game theory (and statistical decision theory) is such that there is no generally accepted prescription of optimal behavior in the non-stationary case; it may very well be that organismic behavior will be found to represent such a solution if and when we understand it. In response to my query on this point, Estes paraphrased the instructions given to his subjects and it seemed significant to me that he had made no effort to suggest to them that they were in fact being confronted with a stationary process. In my own experiments, on the other hand, I had emphasized very strongly the exact nature of the stationary process confronting the subjects* and it had seemed to me that this fact had helped the subjects go to pure strategies. I conjectured, therefore, that the pure type of behavior would be found in subjects who were convinced of stationarity and that the mixed type of behavior would sometimes be found in subjects who believed that there might be non-stationarity. These issues have been much discussed during the Seminar and there seems to be agreement that the matter is worthy of further experimental investigation.

R. R. Bush offered a theoretical explanation of the pure and mixed cases at the initial meeting of one of the four working groups first formed within the Seminar; he based this on the stochastic learning model proposed recently by Bush and Mosteller. Bush did this by showing how the Estes formula is obtained by assigning certain very special values to the parameters in the Bush-Mosteller mathematical model, as later written down in the first WORKING MEMO of the Seminar (WM-1). Bush also derived an asymptotic mixture formula for the r-choice case and, with G. L. Thompson (WM 8), remarked on some of the

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* Even though I stressed this point greatly it was not really accepted by two of my colleagues who, as subjects in one experiment, made inferences from the illustrative example in the written instructions and from apparent patterns of successes and failures [illegible] justified [illegible] were [illegible]

experimental questions involved; the r-choice formula of Bush is:

$$p_i^{\infty} = \frac{\dfrac{1}{1-\pi_i}}{\displaystyle\sum_{j=1}^{r} \dfrac{1}{1-\pi_j}} \qquad \text{for } i = 1, 2, \cdots, r.$$

The punchboard experiments include instances in which r=2 and r=9, so they provide some data that are relevant to the question concerning the applicability of the pure and mixed models discussed by Bush.

Mosteller, in one of the Seminar meetings, presented some experimental results (WM-21) on the 2-choice case taken from recent work of Stanley with rats in mazes and from Jarrett with humans done with a "two-arm bandit" constructed at the instigation of Bush and Mosteller. He discussed the quality of agreement between these experimental data and the pure and mixed models and, among other things, concluded that rats and humans both seem to get to a choice of the better alternative more rapidly when they are rewarded less.

In this memorandum, I shall discuss some punchboard experiments done in the 2-choice and 9-choice cases. In particular, I shall present the results of a pilot experiment done as a result of the Estes talk in an effort to obtain either pure or mixed behavior by human subjects according as they are or are not convinced of non-stationarity in a case that is actually stationary.

2. The experimental method.

The equipment and procedures used are explained in the written instructions given to the subject before the trials start. The set used in the initial 9-choice experiment, where stationarity was stressed, are included here as Appendix A. The set used in the 2-choice experiment, where stationarity was not mentioned, is included in Appendix B. I refer the reader to these two Appendices for information about the equipment and procedure used, and now assume familiarity with this material.

In (A), the 9-choice case, the ten sets of reward probabilities were each chosen at random on the interval (0,1). In (B), the 2-choice case, the ten pairs of reward probabilities were chosen arbitrarily so as to provide information on pairs scattered rather evenly over the possible range; the equipment was identical with that in the 9-choice case, and even the same codes were used, but the subject was limited to choices between a pair of columns specified at the top of each code. The actual values for these pairs are as follows:

Game No.	1	2	3	4	5	6	7	8	9	10
$100\ \pi_1$	51	76	55	40	80	56	98	79	92	96
$100\ \pi_2$	24	09	09	0	30	44	49	59	69	17

In (A), there were two subjects and each did the same ten punchboards with 100 choices on each; one of the two subjects had previously done one punchboard following the instructions of Appendix A; they were both quite unfamiliar with game-theoretic notions.

3. [illegible] Results.

Data are rather meaningless, of course, except as they are interpreted with respect to some well-defined hypothesis. In this instance, our object is to test the general notion that human subjects behave according to a mixed model for response classes that they believe to be [illegible] stationary and they behave according to the pure [illegible] they believe that they have response classes that [illegible]. Specifically, as one way of [illegible] testing the [illegible], we are interested in [illegible] the frequencies [illegible] in each response class with the asymptotic frequencies [illegible] predicted theoretically by the Estes-Bush formulas [illegible] the mixed model.

There is a real difficulty in [illegible] what "[illegible] choices" should mean in this context. [illegible], if [illegible] then the mixed model predicts the [illegible] $\pi_1 < 1$ even though a subject should show very different [illegible] over a large sample of early trials according [illegible] on the one hand, or close to unity on the other. Furthermore, it is quite possible that subjects may behave according to the mixed model for a certain period at the beginning and switch to the pure model after their early experience leads them to the belief that the process is in fact stationary; indeed, it is this hypothesis that I shall consider in the present analysis.

It might be possible to make observations more directly concerning the subject's state of belief with regard to stationarity, perhaps even by asking him after each trial. There is the danger in this, of course, that such queries might alter

the experimental stimulus and thus affect his responses. Some of this danger could be avoided by asking all questions at the close of the work by each subject, and I have sometimes done this. I have nothing systematic to offer as yet on this question, except as some inferences may be made on the basis of the trials data itself. In future experiments, I hope to approach this question more directly.

Some of the principal data for the 2-choice case are presented in Tables 1 and 2 for Subjects PD and MS; these data were obtained using the instructions of Appendix B. Analogous data for the 9-choice case are presented for Subjects 1-10 in Tables 3-12; these data were obtained using the instructions of Appendix A.

TABLE 1

2-Choice (Subject FD)

Code	Column*	Order**	π_1	π_2	p_1	$f_1^{1,100}$	f_1^{1,c_2}	c_2
1	L	4	51	24	61	40	40	100
2	R	9	76	9	79	96	—	4
3	L	5	55	9	66	75	—	25
4	L	1	40	0	63	58	51	86
5	L	6	80	30	78	72	18	34
6	R	10	56	44	56	100	—	--
7	R	2	98	49	96	100	—	--
8	R	3	79	59	66	97	94	52
9	R	7	92	69	80	100	—	--
10	L	8	96	17	95	89	—	11

TABLE 2

2-Choice (Subject MS)

Code	Column*	Order**	π_1	π_2	p_1	$b_1^{1,100}$	f_1^{1,c_2}	c_2
1	L	9	51	24	61	54	39	75
2	R	10	76	9	79	57	[illegible]	83
3	L	1	55	9	66	77	76	94
4	L	7	40	0	63	33	33	100
5	L	2	80	30	78	70	68	95
6	R	6	56	44	56	63	61	95
7	R	3	98	49	96	100	--	—
8	R	5	79	59	66	66	66	100
9	R	4	92	69	80	93	82	38
10	L	8	96	17	95	97	92	36

$p_1^{\infty} = \dfrac{1-\pi_1}{2-\pi_1-\pi_2}$; $f_1^{x,y}$ = percentage of time the π_1 column was chosen during trials x through y; c_2 = trial number when π_2 column last chosen.

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* L means that the π_1 column was on the left and R that it was on the right of the π_2 column.

** This is the order of play for the ten codes

TABLE 3

9-Choice (Subject 1)

Code	Column*	Order**	π_i^{max}	p^{∞}	$f^{1,100}$	$f^{1,c}$	c	$f^{5,100}$
1	8	8	94	57	90	82	57	86
2	8	3	96	55	81	[illegible]	69	[illegible]
3	2	7	91	41	68	65	91	84
4	9	5	100	100	97	0	3	100
5	5	2	92	35	50	42	86	54
6	5	10	84	29	4	04	100	0
7	5	6	98	56	97	0	3	100
8	8	9	98	55	100	0	--	[illegible]
9	8	1	97	57	46	27	74	68
10	9	4	98	58	97	0	3	100

TABLE 4

9-Choice (Subject 2)

Code	Column*	Order**	π_i^{max}	p^{∞}	$f^{1,100}$	$f^{1,c}$	c	$f^{5,100}$
1	8	5	[illegible]	57	81	60	88	100
2	8	2	[illegible]	55	86	86	99	82
3	2	[illegible]	[illegible]	41	63	82	93	98
4	9	1	100	100	78	08	[illegible]	100
5	5	8	[illegible]	[illegible]	0	0	--	0
6	5	[illegible]	84	29	40	40	100	[illegible]
7	5	10	98	56	0	0	100	
8	8	6	[illegible]	55	100	0	--	100
9	8	7		57	100	0	--	100
10	9	9	98	58	[illegible]	0		0

* This is the number of the column for which the probability π_i of a win was greatest.

** This is the order in which the subject played the ten codes.

p^{∞} refers to the column i^* for which π_i was greatest, and is

$$p^{\infty} = \frac{\frac{1}{1-\pi_{i^*}}}{\sum \frac{1}{1-\pi_i}} .$$

$f^{x,y}$ is the percentage of times the column with max π_i was chosen during trials x through y.

c is the trial number after which the column with max π_i was [illegible] not chosen.

TABLE 5

9-Choice (Subject 3)

Code	Column	Order	w_1^{max}	p^∞	$r^{1,100}$	$r^{1,c}$	c	$r^{51,100}$
1	8	8	94	57	26	19	91	52
2	8	3	96	55	22	0	78	44
3	2	4	91	41	30	29	98	60
4	9	5	100	100	54	0	46	100
5	5	6	92	35	14	14	100	8
6	5	9	84	29	28	27	99	46
7	5	7	98	56	14	14	100	8
8	6	1	98	55	14	14	100	14
9	8	2	97	57	92	92	100	86
10	9	10	98	58	90	0	10	100

TABLE 6

9-Choice (Subject 4)

Code	Column	Order	w_1^{max}	p^∞	$r^{1,100}$	$r^{1,c}$	c	$r^{51,100}$
1	8	1	94	57	14	14	100	14
2	8	10	96	55	0	0	---	0
3	2	8	91	41	0	0	---	0
4	9	9	100	100	100	0	---	[illegible]
5	5	3	92	35	7	0	[illegible]	14
6	5	2	84	29	23	23	[illegible]	0
7	5	5	98	56	1	1	100	0
8	8	7	98	55	0	0	100	0
9	8	6	97	57	0	0	100	0
10	9	4	98	58	0	0	100	0

TABLE 7

9-Choice (Subject 5)

Code	Column	Order	W_1^{max}	p^{∞}	$f^{1,100}$	$f^{1,c}$	e	$f^{51,100}$
1	8	6	94	57	77	0	23	100
2	8	2	96	55	0	0	100	0
3	2	5	91	41	0	0	100	0
4	9	3	100	100	57	0	43	100
5	5	4	92	35	88	0	12	100
6	5	1	84	29	15	15	100	16
7	5	7	98	56	0	0	100	0
8	8	9	98	55	100	0	—	100
9	6	8	97	57	100	0	—	100
10	9	10	98	58	83	0	17	100

TABLE 8

9-Choice (Subject 6)

Code	Column	Order	W_1^{max}	p^{∞}	$f^{1,100}$	$f^{1,c}$	e	$f^{51,100}$
1	8	2	94	57	74	43	51	98
2	8	7	96	55	0	0	100	0
3	2	1	91	41	68	16	38	100
4	9	10	100	100	100	0	—	100
5	5	8	92	35	0	0	100	0
6	5	6	84	29	60	49	39	100
7	5	5	98	56	92	0	8	100
8	8	3	98	55	0	0	100	0
9	8	4	97	57	0	0	100	0
10	9	9	98	58	90	0	10	100

TABLE 9

9-Choice (Subject 7)

Code	Column	Order	W_1^{max}	p^{∞}	$f^{1,100}$	$f^{1,c}$	c	$f^{51,100}$
1	8	7	94	57	96	95	86	96
2	8	5	96	55	96	96	89	94
3	2	2	91	41	12	12	100	6
4	9	10	100	100	100	0	—	100
5	5	3	92	35	0	0	100	0
6	5	9	84	29	50	4	52	96
7	5	6	98	56	0	0	100	0
8	8	1	98	55	11	11	100	14
9	8	4	97	57	100	0	—	100
10	9	8	98	58	98	0	2	100

TABLE 10

9-Choice (Subject 8)

Code	Column	Order	W_1^{max}	p^{∞}	$f^{1,100}$	$f^{1,c}$	c	$f^{51,100}$
1	8	9	94	57	100	0	—	100
2	8	5	96	55	0	0	100	0
3	2	6	91	41	85	40	25	100
4	9	7	100	100	0	0	100	0
5	5	1	92	35	25	25	100	30
6	5	3	84	29	85	0	15	100
7	5	2	98	56	48	48	100	48
8	8	8	98	55	0	0	100	0
9	8	4	97	57	0	0	100	0
10	9	9	98	58	100	0	—	100

TABLE 11

(9-Choice (Subject 9)

Code	Column	Order	$\max v_1$	p^{∞}	$r^{1,100}$	$r^{1,c}$	c	$r^{51,100}$
1	8	1	94	57	12	12	100	12
2	6	5	96	55	16	16	100	12
3	2	8	91	41	0	0	100	0
4	9	3	100	100	6	6	100	6
5	5	9	92	35	5	5	100	10
6	5	2	84	29	12	12	100	12
7	5	4	98	56	36	36	100	0
8	8	10	98	55	30	7	75	52
9	8	7	97	57	3	2	99	6
10	9	6	98	58	15	15	100	24

TABLE 12

9-Choice (Subject 10)

Code	Column	Order	$\max v_1$	p^{∞}	$r^{1,100}$	$r^{1,c}$	c	$r^{51,100}$
1	8	2	94	57	75	70	82	74
2	8	8	96	55	0	0	100	0
3	2	10	91	41	61	3	40	100
4	9	3	100	100	88	0	12	100
5	5	5	92	35	67	62	86	68
6	5	6	84	29	64	6	39	100
7	5	4	98	56	3	0	97	6
8	8	9	98	55	0	0	100	0
9	8	7	97	57	95	94	89	94
10	9	1	98	58	24	19	94	44

It seems very unlikely that the choices of Subject FD can be explained well in terms of the mixed model. For example, on his last board all his choices were made on a column that gave wins on only 56 per cent of the trials. One possible explanation of this behavior is that he set a standard of 50 per cent wins as satisfactory and then did not search for a better column as long as he felt that this standard was met, as was the case on this last board after the sixth trial. A fact consistent with this explanation is his continued effort (42 attempts through trial 86) on his first board to win on a column that never paid off while the better column was paying off less than half of the time. These possibilities are typical of the experimental complications that arise because of the particular psychological set that comes with the subject at the outset.

Subject MS, on the other hand, seems to have performed in a manner quite consistent with the mixed model, as judged by the close agreement between p_1^{∞} and $f_1^{1,100}$; his seventh board is a notable exception and it is a surprising fact, in this connection, that he devoted his last 28 trials to a choice that always failed and that had always failed on his previous 39 trials with it. It is significant that he kept trying both columns well through all but two of his boards, and that he remarked during and after the experiment that he was convinced that there was some sort of pattern and that he might find it if he kept on hunting for such regularities. If we suppose that this hunting ended soon after trial c_2 then we would expect that the f_1^{1,c_2} might be in even better agreement with the p_1^{∞} than are the

$f_i^{1,100}$; this seems to be quite true for Subject MS, except for his ninth board, and is further evidence of consistency between his behavior and the notion that the mixed model applies until the subject believes that the process is stationary.

The data on Subjects 1-10 are rather hard to interpret in light of our central hypothesis. Certainly, there is no very striking agreement between p^{∞} and $f^{1,100}$. It does seem to be the usual case that the subject is still choosing columns other than the best one through most of the 100 trials in each game, and the subjects usually fail to settle down on the very best choice within their hundred trials. Of course, it may reasonably be argued that the early trials in each game should not be taken too seriously in an investigation of asymptotic behavior; for this reason, I have included the $f^{51,100}$ data but the interpretation is difficult here, too.

One source of trouble in the analysis is the similarity between the π_i in some cases, so that it is hard for the subjects to discriminate between such columns. For example, in Code 7 the three largest π_i are 98, 95, and 90 so that it would not be at all surprising if a subject were to spend some of his choices on the 95 per cent column that the mixed model would allocate to the 98 per cent and 90 per cent column. Consequently, in an effort to soften this confusing effect, the data were regrouped for analysis by combining columns with comparable π_i values. The rule for this regrouping was quite arbitrary. It consisted in combining all data for those columns

whose π_1 values differed by 7 per cent or less from the max π_1 into a new column 1, then similarly forming a new column 2 to include all those columns whose π_1 values differed by 7 per cent or less from the max π_1 not already included in the new column 1 and continuing until all the columns were regrouped. If we let r^j denote the number of new columns formed for code j, and let π_k^j, for $k=1,2,\cdots, r^j$, denote the mean of the set of π_1 values represented in the new column k of code j, then the mixed model requires that the asymptotic frequencies of play on new column k for code j are given by the following expressions for p_k^j:

$$p_k^j = \frac{\dfrac{1}{1-\pi_k^j}}{\displaystyle\sum_{\alpha=1}^{r^j} \frac{1}{1-\pi_\alpha^j}} \qquad \text{for } j=1,2,\cdots, 10;\ k=1,2,\cdots, r^j.$$

The regrouped data are presented and compared with the values so obtained for p_k^j in Tables 13-22.

TABLE 13

r^1-Choice (Code 1)

Subject	f^1_1	f^1_2	f^1_3	f^1_4	f^1_5	f^{51}_1	f^{51}_2	f^{51}_3	f^{51}_4	f^{51}_5
1	90	5	2	2	1	86	6	4	2	2
2	81	14	2	1	2	100	0	0	0	0
3	26	40	10	14	10	52	40	0	8	0
4	14	60	9	12	5	14	64	8	10	4
5	77	11	3	6	3	100	0	0	0	0
6	74	18	3	3	2	100	0	0	0	0
7	96	0	0	4	0	96	0	0	4	0
8	100	0	0	0	0	100	0	0	0	0
10	75	13	2	7	3	74	20	2	0	4
Average*	70.4	17.9	3.4	5.4	2.9	80.2	14.4	1.6	2.7	1.1
p^1_k	74	8	7	6	5	74	8	7	6	5

TABLE 14

r^1-Choice (Code 2)

Subject	f^1_1	f^1_2	f^1_3	f^1_4	f^1_5	f^{51}_1	f^{51}_2	f^{51}_3	f^{51}_4	f^{51}_5
1	81	10	5	2	2	98	2	0	0	0
2	86	10	0	2	2	82	14	0	2	2
3	22	52	10	9	7	44	36	0	8	12
4	0	97	2	0	1	0	100	0	0	0
5	0	89	0	6	5	0	98	0	2	0
6	0	48	46	4	2	0	88	4	6	2
7	96	4	0	0	0	94	6	0	0	0
8	0	0	0	100	0	0	0	0	100	0
10	0	96	0	0	4	0	96	0	0	4
Average	31.7	45.1	7.0	13.7	2.5	35.4	48.9	.4	13.1	2.2
p^1_k	67	14	11	5	3	67	14	11	5	3

* Subject 9 was excluded in computing the averages because his selections differed so greatly from all the others.

f^x_k is the percentage of time that [illegible] column k was chosen during trials x through 100.

TABLE 15

r'-Choice (Code 3)

Subject	r_1^1	r_2^1	r_3^1	r_4^1	r_5^1	r_6^1	r_1^{51}	r_2^{51}	r_3^{51}	r_4^{51}	r_5^{51}	r_6^{51}
1	68	18	9	1	2	2	84	8	8	0	0	0
2	83	6	2	1	5	3	98	0	2	0	0	0
3	30	23	21	4	14	8	60	0	22	0	14	4
4	0	85	0	10	0	5	0	100	0	0	0	0
5	0	97	0	0	3	0	0	100	0	0	0	0
6	68	6	9	3	8	6	100	0	0	0	0	0
7	12	28	9	11	23	17	6	34	8	14	24	14
8	85	0	10	0	5	0	100	0	0	0	0	0
10	61	24	4	2	6	3	100	0	0	0	0	0
Average	45.3	31.8	7.1	3.6	7.3	4.9	60.9	26.9	4.4	1.6	4.2	2.0
p_k^1	52	20	11	7	5	5	52	20	11	7	5	5

TABLE 16

r'-Choice (Code 4)

Subject	r_1^1	r_2^1	r_3^1	r_4^1	r_5^1	r_6^1	r_1^{51}	r_2^{51}	r_3^{51}	r_4^{51}	r_5^{51}	r_6^{51}
1	97	3	0	0	0	0	100	0	0	0	0	0
2	78	3	4	6	6	3	100	0	0	0	0	0
3	54	6	12	11	11	6	100	0	0	0	0	0
4	100	0	0	0	0	0	100	0	0	0	0	0
5	57	6	7	12	12	6	100	0	0	0	0	0
6	100	0	0	0	0	0	100	0	0	0	0	0
7	100	0	0	0	0	0	100	0	0	0	0	0
8	0	80	10	0	5	5	0	78	[illegible]	0	4	8
10	88	1	0	0	0	0	100	0	0	0	0	0
Average	74.9	12.2	3.7	3.2	3.8	[illegible]	88.9	[illegible]	1.1	0	[illegible]	[illegible]
p_k	100	0	0	0	0	0	100	0	0	0	0	0

TABLE 17

r'-Choice (Code 5)

Subject	f_1^1	f_2^1	f_3^1	f_4^1	f_5^1	f_6^1	f_1^{51}	f_2^{51}	f_3^{51}	f_4^{51}	f_5^{51}	f_6^{51}
1	86	2	3	6	2	1	100	0	0	0	[illegible]	0
2	94	0	0	6	0	0	94	0	0	6	[illegible]	0
3	25	10	21	11	22	11	30	0	22	22	24	2
4	7	69	20	0	2	2	14	54	28	0	[illegible]	2
5	88	3	3	0	3	3	100	0	0	0	[illegible]	0
6	91	4	3	2	0	0	100	0	0	0	[illegible]	0
7	81	0	6	10	2	1	84	0	2	14	[illegible]	0
8	27	43	13	3	8	6	30	52	10	0	[illegible]	4
10	77	0	13	5	1	4	90	0	0	10	[illegible]	0
Average	64.0	14.6	9.1	4.8	4.4	3.1	71.3	11.8	6.9	5.8	[illegible]	.9
F_k	47	24	9	8	7	5	47	24	9	8		5

TABLE 18

r'-Choice (Code 6)

Subject	f_1^1	f_2^1	f_3^1	f_4^1	f_5^1	f_6^1	f_1^{51}	f_2^{51}	f_3^{51}	f_4^{51}	f_5^{51}	f_6^{51}
1	4	79	3	5	3	6	0	96	0	4	0	0
2	40	43	1	9	1	6	50	50	0	0	0	0
3	28	37	4	5	10	16	46	54	0	0	[illegible]	0
4	23	51	0	3	6	17	0	[illegible]	0	4	[illegible]	26
5	15	27	[illegible]	10	10	31	16	30	6	12	8	28
6	80	14	1	1	0	4	100	0	0	0	[illegible]	0
7	[illegible]	28	10	4	2	6	[illegible]	4	0	0	[illegible]	0
8	85	2	1	1	1	10	100	0	0	0	[illegible]	0
10	64	7	10	6	2	11	100	0	0	0	[illegible]	0
Average	[illegible]	32.0	4.1	4.9	3.9	[illegible]	[illegible]	[illegible]	.7	2.2	[illegible]	6.0
F_k	39	21	14	11	8	7	39	21	14	11	[illegible]	7

TABLE 19

r'-Choice (Code 7)

Subject	f_1^1	f_2^1	f_3^1	f_4^1	f_5^1	f_6^1	f_1^{51}	f_2^{51}	f_3^{51}	f_4^{51}	f_5^{51}
1	99	0	0	0	1	0	100	0	0	0	0
2	98	0	0	0	0	2	100	0	0	0	0
3	33	13	10	20	14	10	46	6	0	20	28
4	1	72	0	27	0	0	2	96	0	2	0
5	97	0	0	0	3	0	100	0	0	0	0
6	92	1	2	3	1	1	100	0	0	0	0
7	99	0	0	0	1	0	100	0	0	0	0
8	48	0	52	0	0	0	48	0	52	0	0
10	4	93	0	0	3	0	6	94	0	0	0
Average	63.4	19.9	7.1	5.6	2.6	1.4	66.9	21.8	5.8	2.4	3.1
p_k^1	63	22	6	4	3	2	63	22	6	4	3

TABLE 20

r'-Choice (Code 8)

Subject	f_1^1	f_2^1	f_3^1	f_4^1	f_1'	f_2'	f_3^{51}	f_4^{51}
1	100	0	0	0	100	0	0	0
2	100	0	0	0	100	0	0	0
3	41	15	27	17	50	20	18	12
4	100	0	0	0	100	0	0	0
5	100	0	0	0	100	0	0	0
6	100	0	0	0	100	0	0	0
7	25	19	46	10	30	20	4[illegible]	8
8	100	0	0	0	100	0	0	0
10	96	0	0	4	92	0	0	8
Average	84.7	3.8	8.1	3.4	85.8	4.4	6.7	3.1
p_k^2	68	18	10	4	68	18	10	4

TABLE 21

r′-Choice (Code 9)

Subject	r^1_1	r^1_2	r^1_3	r^1_4	r^1_5	r^{51}_1	r^{51}_2	r^{51}_3	r^{51}_4	r^{51}_5
1	75	3	3	10	9	100	0	0	0	0
2	100	0	0	0	0	100	0	0	0	0
3	96	0	0	2	2	94	0	0	4	0
4	98	0	0	0	2	98	0	0	0	2
6	100	0	0	0	0	100	0	0	0	0
7	100	0	0	0	0	100	0	0	0	0
8	100	0	0	0	0	100	0	0	0	0
10	95	0	0	0	5	94	0	0	0	6
Average	96.0	.3	.3	1.4	2.0	98.4	0	0	.4	[illegible]
p^1_k	69	12	8	6	5	69	12	8	6	5

TABLE 22

r′-Choice (Code 10)

Subject	r^1_1	r^1_2	r^1_3	r^1_4	r^1_5	r^{51}_1	r^{51}_2	r^{51}_3	r^{51}_4	r^{51}_5
1	97	1	0	1	1	100	0	0	0	0
2	99	1	0	0	0	100	0	0	0	0
3	92	2	4	1	1	100	0	0	0	0
4	99	1	0	0	0	100	0	0	0	0
5	83	14	0	0	3	100	0	0	0	0
6	90	2	6	1	1	100	0	0	0	0
7	98	2	0	0	0	100	0	0	0	0
8	100	0	0	0	0	100	0	0	0	0
10	33	14	31	13	9	50	14	22	6	8
Average	87.9	4.1	4.6	1.8	1.6	94.4	1.6	2.4	.7	9
p^1_k	85	6	3	3	3	85	6	3	3	3

It would be a stretch on the imagination to argue that Tables 13–22 provide evidence in support of the hypothesis that our subjects behaved according to the mixed model, but a still greater stretch to argue that these data permit us to reject the hypothesis. My own view is that a much more extensive experiment and a better-conducted one, is necessary before any real conclusions can be drawn. I believe that the most promising approach is to start with the 3-choice case in which the π_s are rather evenly spaced, and not too close to 0 or 1, in order to check the closeness of agreement with the Estes–Bush formulas for p_i. Not until experimental techniques are good enough to give constant and repeatable results in agreement for the 3-choice case with either the mixed or pure models, at will, would I want to tackle the problem of determining the precise nature of the experimental stimulus that is necessary to produce this kind of difference in behavior—and only after I understood this stimulus problem reasonably well would I again want to work with the r-choice case for $r > 3$. The crude pilot experiments reported here may be of some use to others participating in the Seminar who have become interested in this problem of the pure vs. mixed learning behavior.

APPENDIX A

INSTRUCTIONS FOR STATIC-NINE EXPERIMENT

A. Static-Nine

The equipment for static-nine consists of a "punchboard," a "punch," a "code," and a "register." The code and register are printed forms fastened to the back and the front, respectively, of the punchboard. The punch is used on each move to make a hole in the register signifying the choice of an integer from 1 through 9. Samples of code and register are attached to these instructions.

The register has 25 lines, and in each line the integers 1 through 9 appear in four "fields" across the page; this provides for 100 moves.

The code has either a 1 or a 0 in each position. A 1 denotes a win and a 0 denotes a loss. The code is arranged so that the mark 1 appears a preassigned number of times in each column. The order of the marks 1 and 0 in each column is random.

The first move is made by punching out one of the nine digits in row 1 of field 1. If a 1 is seen through the hole this position is circled in pencil by the player to denote a win, otherwise it is left uncircled to denote a loss. The second move is made similarly by punching in row 2 of field 1, and circling to denote a 1 if observed. After the 25 moves in field 1 are completed, start at the top of field 2, and continue in this way until all 100 rows have been punched. The score on these 100 moves is the total number of circled positions.

After you have made 100 trials, you will give the umpire instructions for your plays in the next hundred trials. You do this by assigning to each choice, 1 through 9, a number indicating how often you wish that choice played in the next 100 trials. The nine numbers must add to 100. For example, you might assign 20, 32, 13, 10, 25 to digits 1, 3, 7, 8, 9 respectively, and zeros to the others, indicating that you want digit 1 played 20 times, digit 2 played no times, digit played 32 times, etc. In particular, if you wish to have some one number (say 5) played all the time, then you would assign 100 to that one and zero to each of the others. These numbers should be written in the row provided on the data form for this purpose.

Your object is to get as many wins as you can in the 200 moves

B. General Information

This is an experiment designed to obtain a quantitative comparison of the ability of people with that of rats in learning to play a certain simple intellectual game. Bush and Mosteller, at Harvard, have examined experimental data obtained by psychologists in their studies of rat learning. They have developed a mathematical model that seems to fit the rat data quite well. I am using this model, which they have called the "stat rat," to compute the probable performance of rats in playing von Neumann-Morgenstern games. The scientific purpose is to test the validity of various mathematical theories of learning and decision.

The punchboard represents a mechanical umpire who determines wins and losses in the following manner. The umpire first chooses nine numbers between 0 and 1 (for example, .500 or .378) from a random number table; these are thought of as probabilities G_1, G_2, ..., G_9 that your choices of 1 through 9 will win. Each time you play a number (say 3) the umpire determines whether or not you have won by applying the corresponding probability (G_3 in this case) to make the decision; he again uses a random number table for each of these decisions. Thus, if the probability G_3 that your choice 3 would win was .700, you might expect to win 7 times out of 10 when you choose the number 3.

For the second hundred trials the umpire simply multiplies the nine numbers written on your data form by the corresponding probabilities G_1, G_2, ..., G_9, and adds these products together to get your total of wins on the second hundred trials. Of course, this computational procedure produces the same result as the one that you would expect to get if the umpire actually went through the second hundred plays for you one by one.

We should like to have you play static-nine several times. Your average score for all your games will be compared with that of the stat-rat, and those of the other subjects in this experiment. You will be told the stat-rat's score after each game. At the conclusion of the experiment, I will send each subject a recapitulation of the names and scores of all players, including the stat-rat.

Thank you for participating, and good luck.

SAMPLE DATA FORM

Name____ Date______________

Game Number	Frequency for Choice Number:								
	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									

BLANK PAGE

SAMPLE REGISTER

1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	2	[illegible]	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	[illegible]	8	9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	[illegible]
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	[illegible]
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	6	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	[illegible]	[illegible]
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	5	[illegible]
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

APPENDIX B

INSTRUCTIONS

Static-nine

The equipment for static-nine consists of a "punchboard," a "punch," a "code," and a "register." The code and register are printed forms fastened to the back and the front, respectively, of the punchboard. The punch is used on each move to make a hole in the register signifying the choice of an integer from 1 through 9. Samples of code and register are attached to these instructions.*

The register has 25 lines, and in each line the integers 1 through 9 appear in four "fields" across the page; this provides for 100 moves. At the top of each register you will find two digits written in red; you are to punch only one or the other of these two on each trial.

The code has either a 1 or a 0 in each position. A 1 denotes a win and a 0 denotes a loss.

The first move is made by punching out one of two digits in row 1 of field 1. If a 1 is seen through the hole this position is circled in pencil by the player to denote a win; otherwise it is left uncircled to denote a loss. The second move is made similarly by punching one of the two digits in row 2 of field 1, and circling to denote a 1 if observed. After the 25 moves in field 1 are completed, start at the top of field 2, and continue in this way until all 100 rows have been punched. The score on these 100 moves is the total number of circled positions.

* See samples in Appendix A.

Your object is to get as many wins as you can in the 100 moves.

General Information

This is an experiment designed to obtain a quantitative comparison of the ability of people with that of rats in learning to play a certain simple intellectual game. Bush and Mosteller, at Harvard, have examined experimental data obtained by psychologists in their studies of rat learning. They have developed a mathematical model that seems to fit the rat data quite well. I am using this model, which they have called the "stat-rat," to compute the probable performance of rats in playing von Neumann-Morgenstern games. The scientific purpose is to test the validity of various mathematical theories of learning and decision.

We should like to have you play static-nine several times. Your average score for all your games will be compared with that of the stat-rat, and those of the other subjects in this experiment. At the conclusion of the experiment, I will be glad to give you a recapitulation of the names and scores of other players, including the stat-rat, if you wish.

Thank you for participating, and good luck.